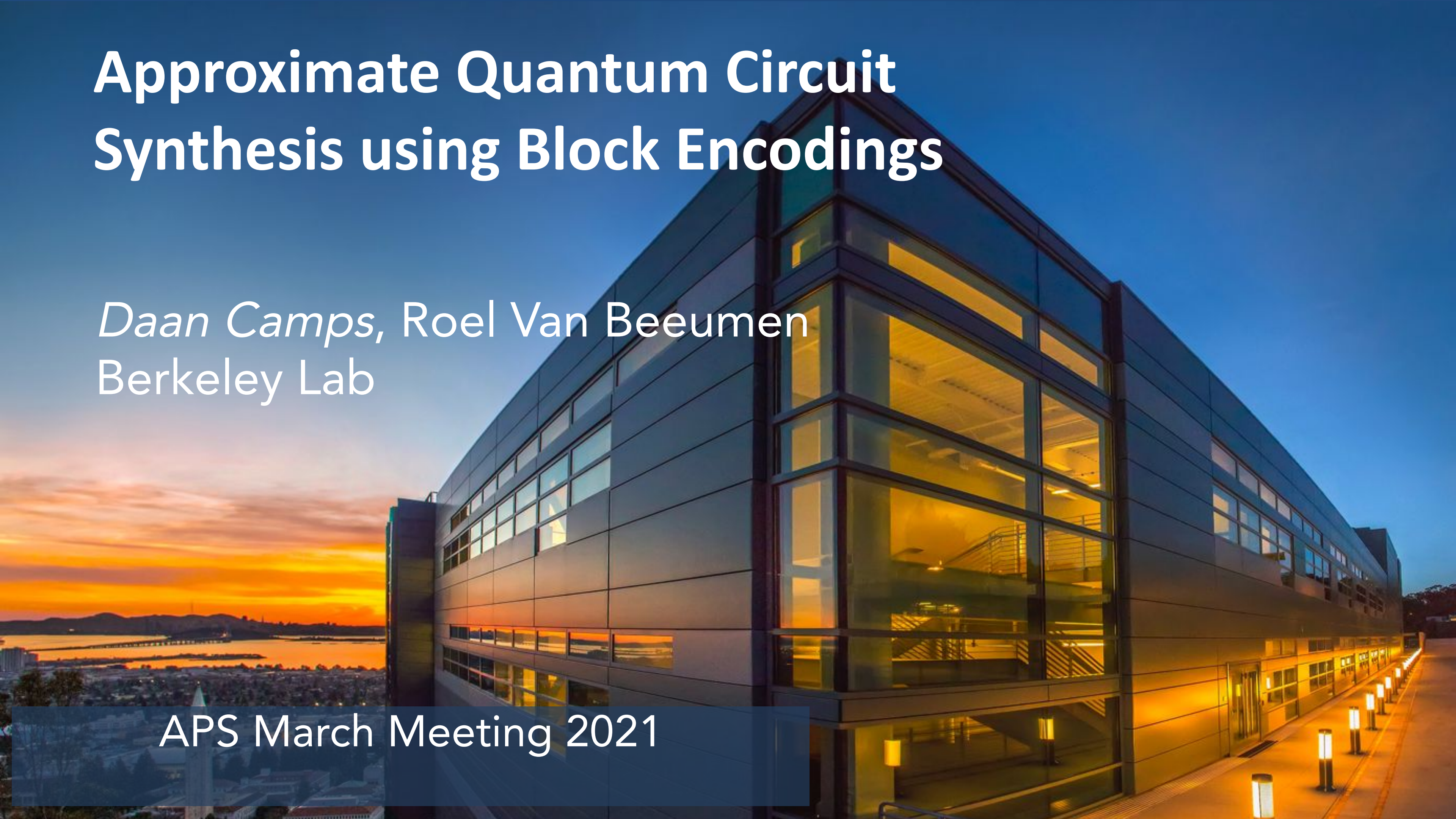

Approximate Quantum Circuit Synthesis using Block Encodings

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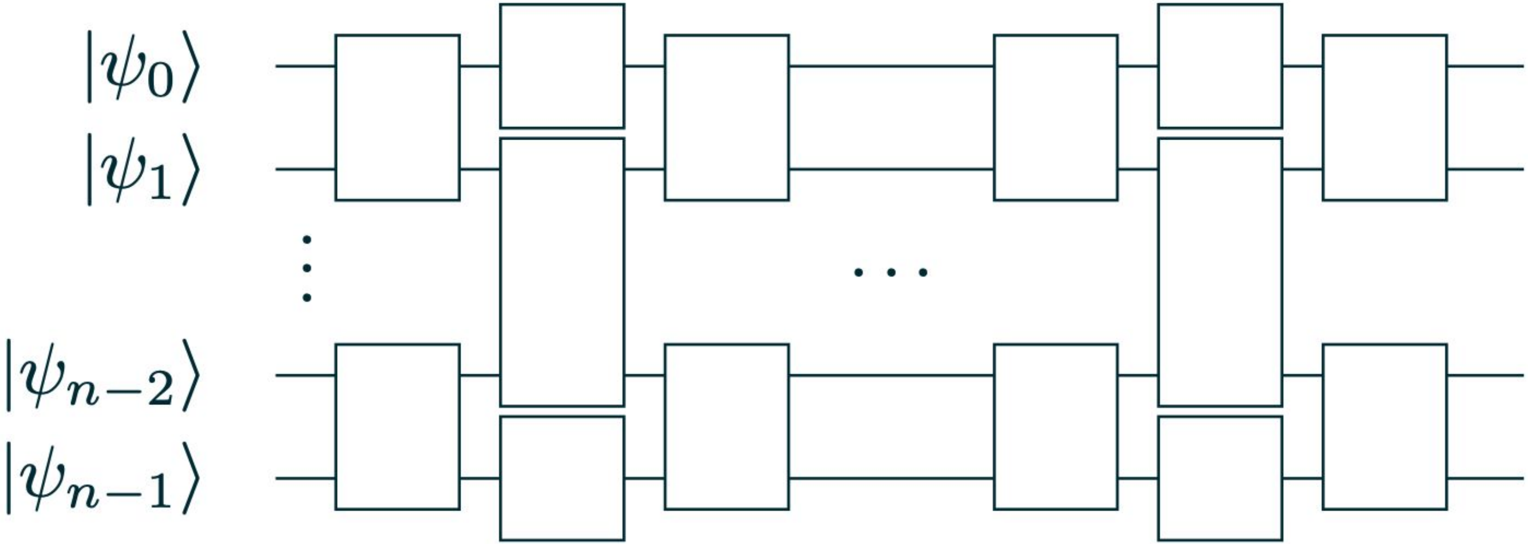
Berkeley Lab

APS March Meeting 2021

Quantum algorithms: unitaries with efficient quantum circuits



Synthesis: A well studied subject with many different approaches

Algebraic Techniques

Cosine-Sine Decomposition

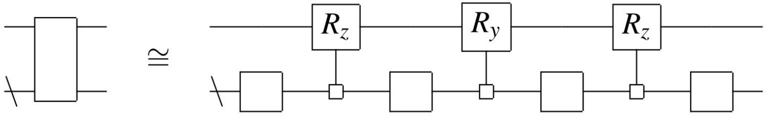


Image: Shende, Bullock, Markov (2006)

KAK Decomposition

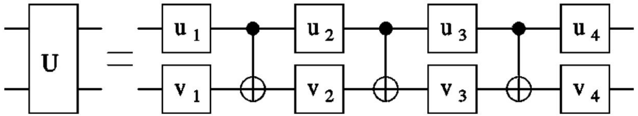


Image: Vidal, Dawson (2004)

Givens QR Decomposition

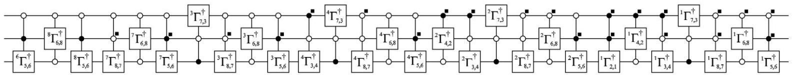


Image: Vartiainen, Mötiönen, Salomaa (2004)

Optimization Techniques

QFAST

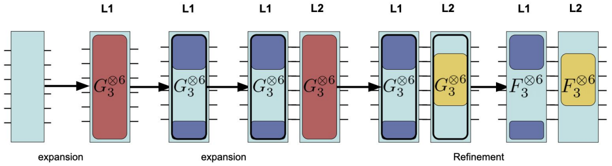


Image: Younis, Sen, Yelick, Iancu (2020)

Repeat-Until-Success Techniques

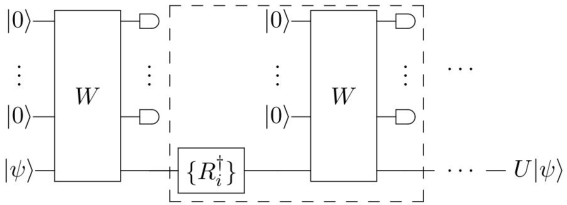
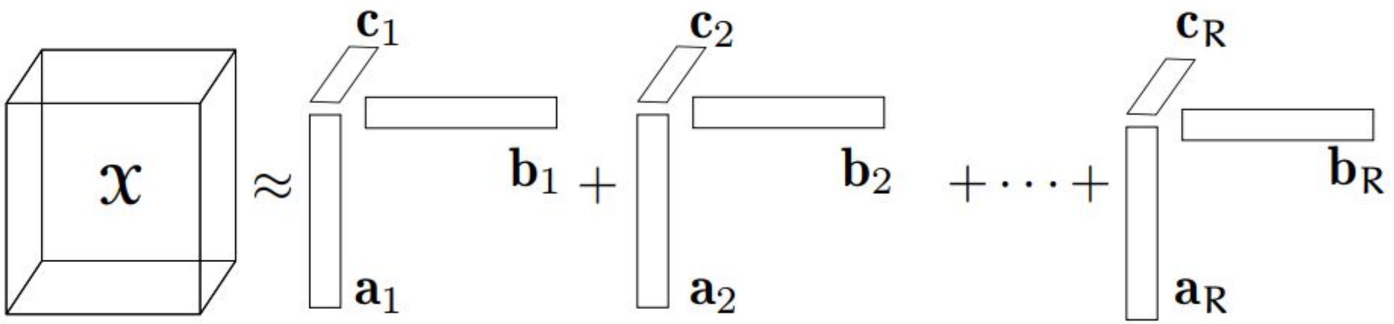


Image: Paetznick, Svore (2014)

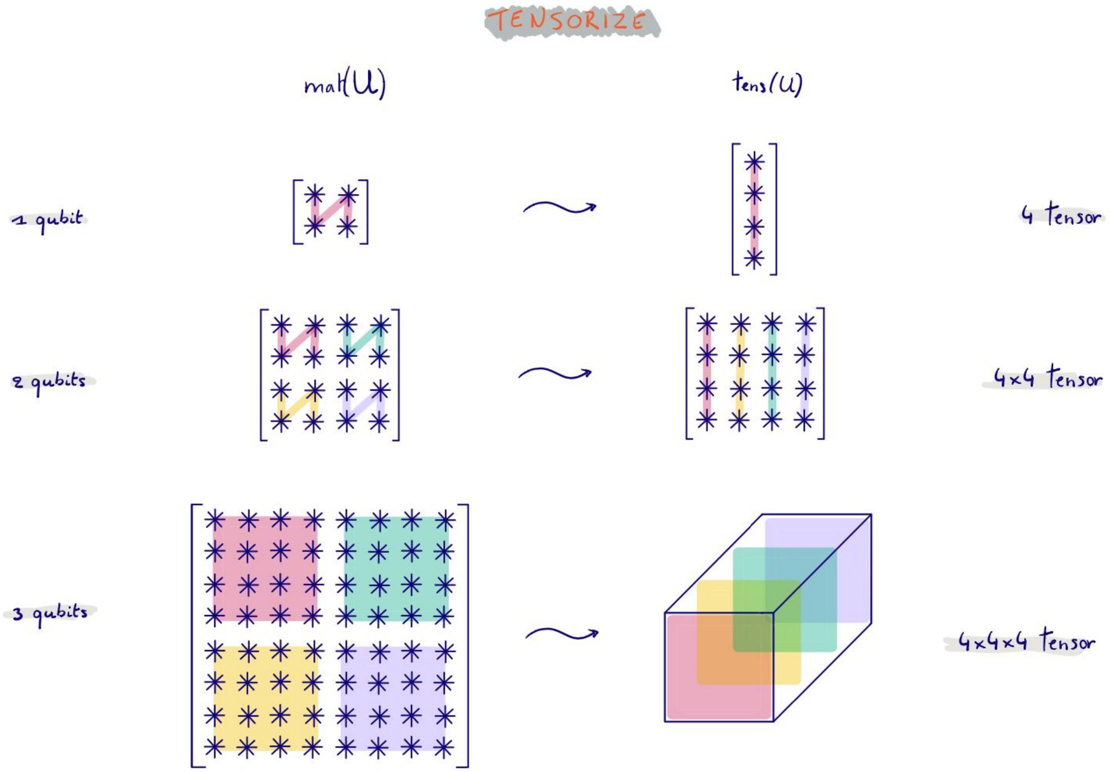
Computational tool from numerical linear algebra

Tensor Rank Decomposition



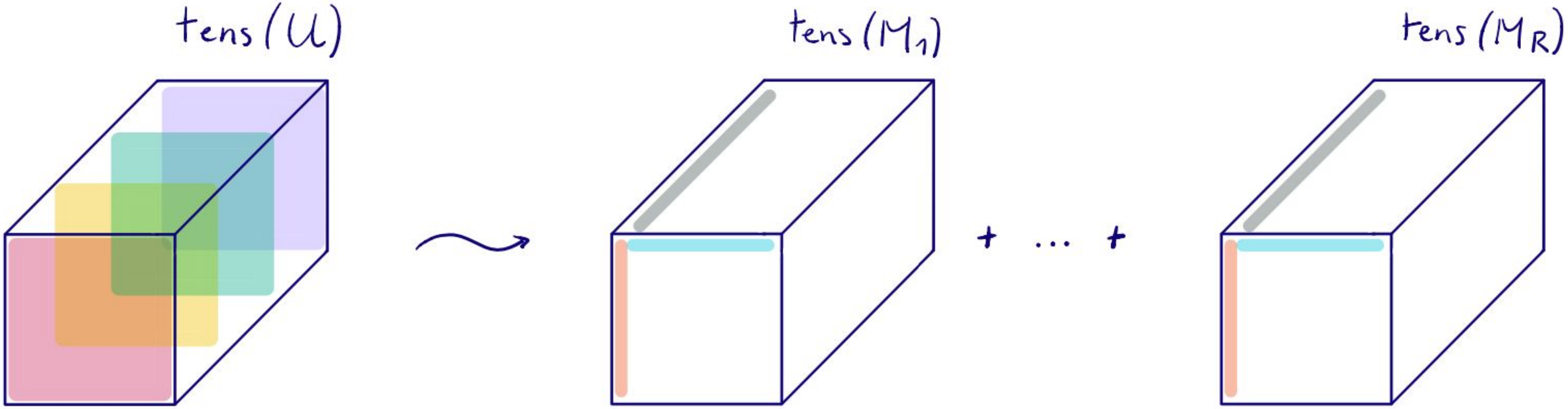
- Widely used in:
  - numerical linear algebra
  - scientific computing
  - data analysis
- Uniqueness
- Good optimization algorithms

Tensorizing the unitary



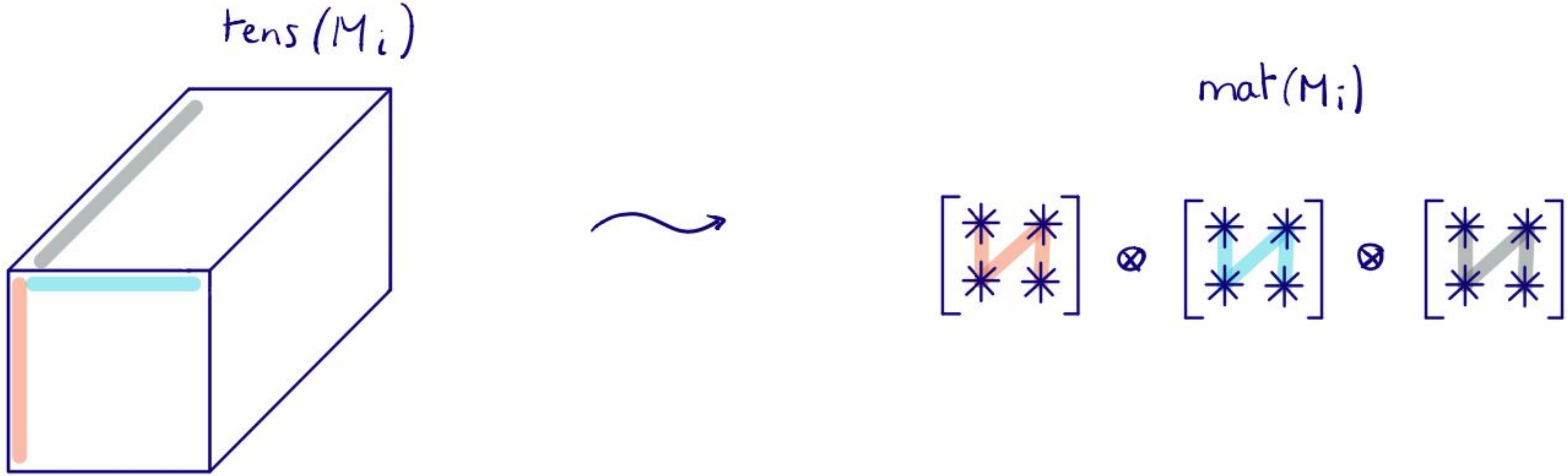
Decompose the tensor

DECOMPOSE TENSOR IN RANK-1 TERMS

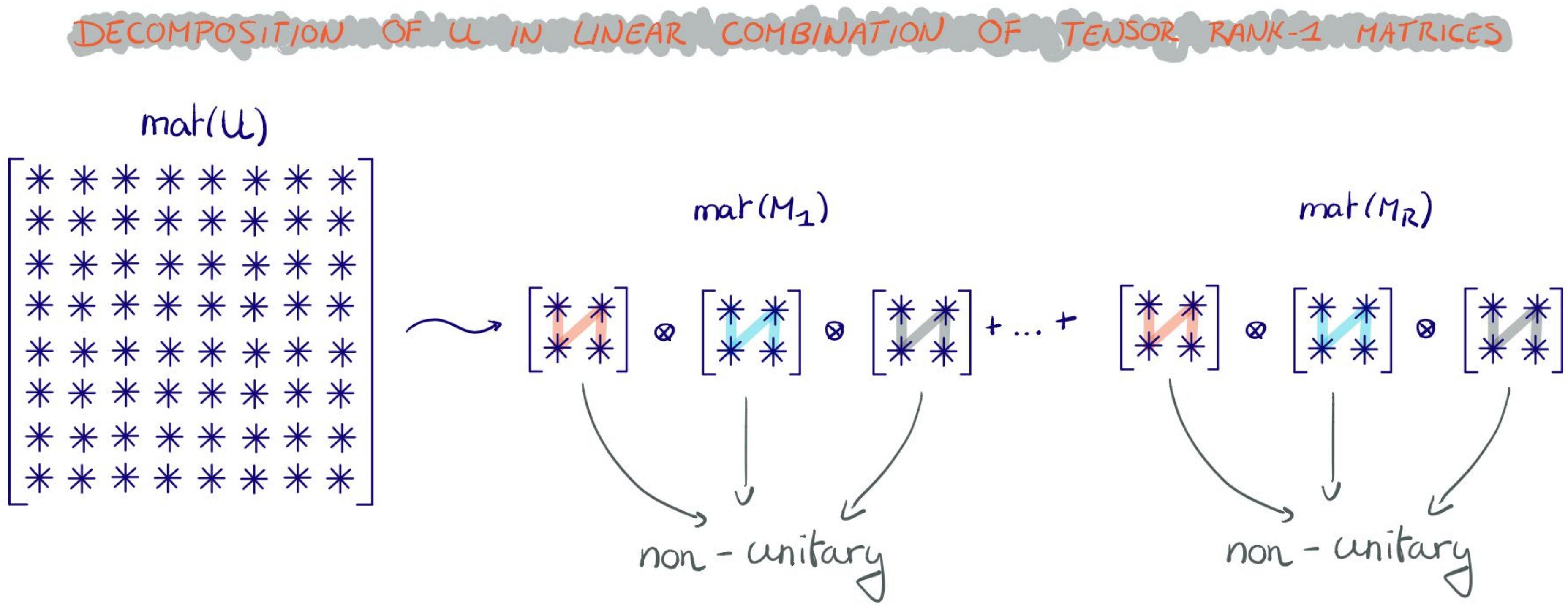


Matricizing the rank-1 tensors

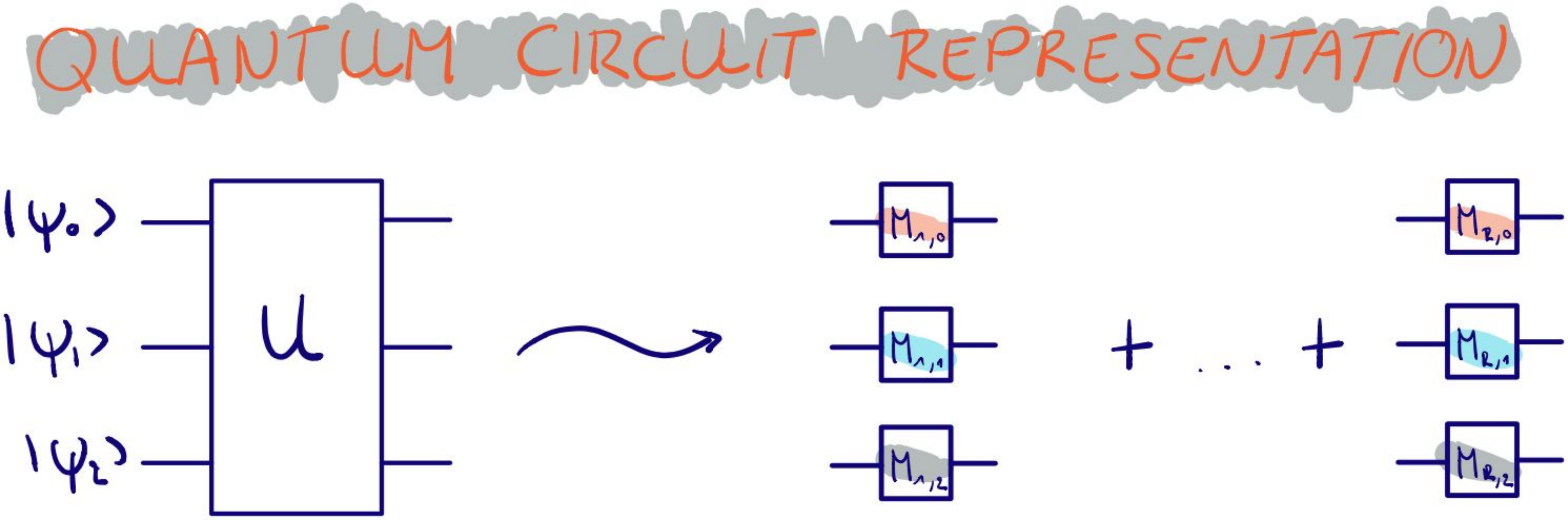
MATRICIZE RANK-1 TENSORS



Unitary decomposed in tensor rank-1 matrices

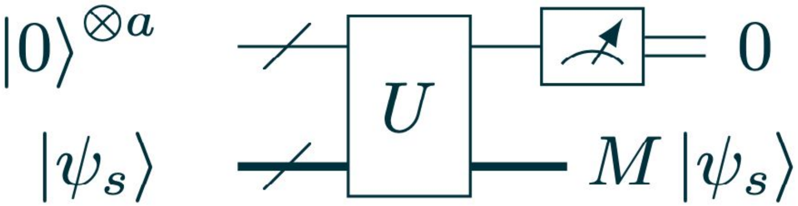


More or less a quantum circuit diagram



Relaxing unitary constraints using block encodings [Gilyen et al, 2018]

$$M \longrightarrow U := \begin{bmatrix} M & * \\ * & * \end{bmatrix} \longrightarrow$$



- Embed the non-unitary matrix M in a larger unitary matrix U
- Distinction between additional *ancilla* qubits and *signal* qubits
- Measurement of *ancilla* qubits
  - Probabilistic implementation of M, similar to Repeat-Until-Success strategy
- Amplitude amplification

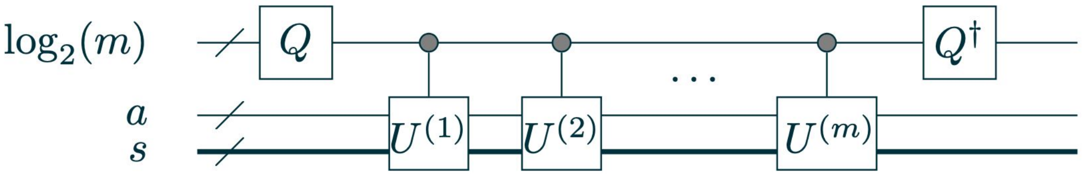
Combining block encodings in tensor products

$$M^{(1)}$$
$$\downarrow$$
$$\begin{bmatrix} M^{(1)} & * \\ * & * \end{bmatrix}$$
$$\downarrow$$
$$U^{(1)}$$

Sums of block encodings [Childs & Wiebe, 2012]

$$y_1 M^{(1)} \quad + \quad y_2 M^{(2)} \quad + \quad \cdots \quad + \quad y_m M^{(m)}$$

$$U^{(i)} = \begin{bmatrix} M^{(i)} & * \\ * & * \end{bmatrix}$$

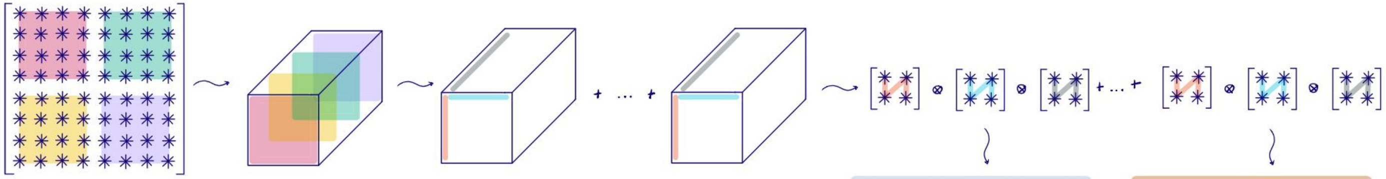


$$Q = \frac{1}{\|y\|_1} \begin{bmatrix} \sqrt{y_1} & * & \cdots & * \\ \sqrt{y_2} & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{y_m} & * & \cdots & * \end{bmatrix}$$

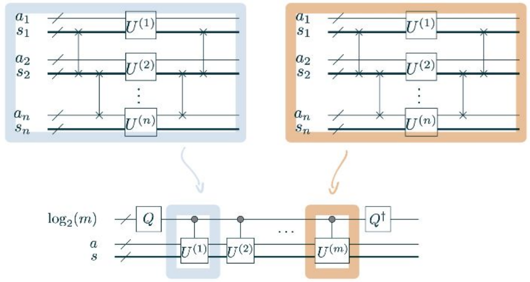
$$\downarrow$$

$$U = \begin{bmatrix} y_1 M^{(1)} + y_2 M^{(2)} + \cdots + y_m M^{(m)} & * \\ * & * \end{bmatrix}$$

Bringing it together



$$U = \begin{bmatrix} \sum_{i=1}^{m} y_i M^{(i,1)} \otimes M^{(i,2)} \otimes \cdots \otimes M^{(i,n_i)} & * \\ * & * \end{bmatrix}$$



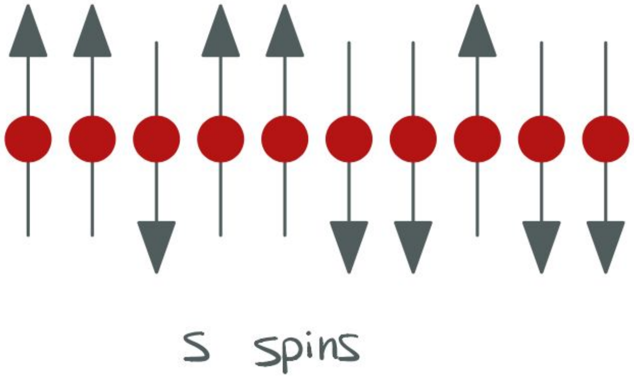
This circuit construction has a sub-exponential efficient gate complexity if:

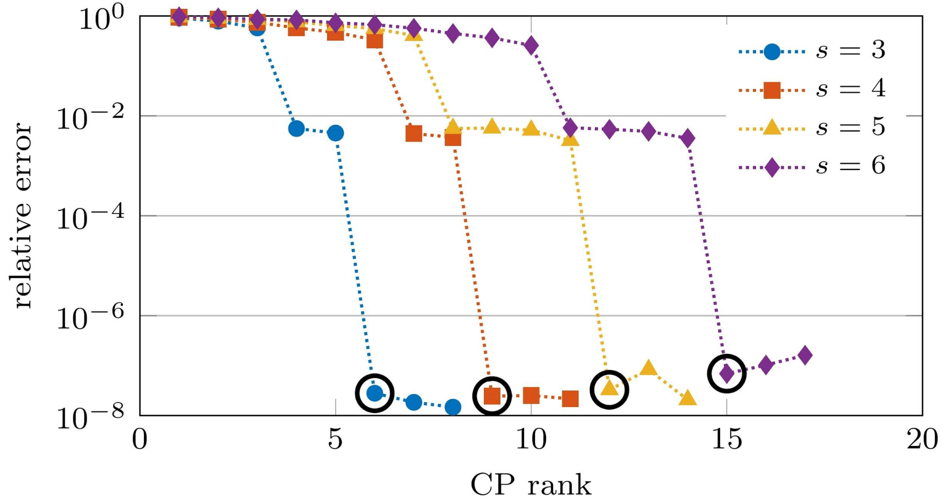
- the tensor rank is sub-exponential
- efficient circuits exist for the individual block encodings

Localized Hamiltonians have low-rank tensor structure

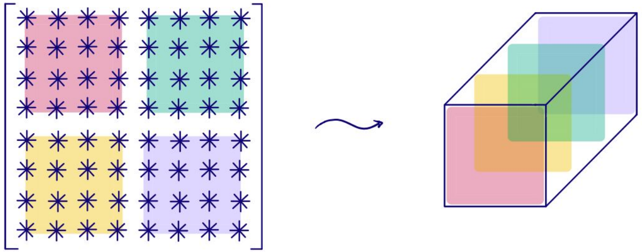
Heisenberg XYZ Hamiltonian

$$H_{\text{XYZ}} = \sum_{i=1}^{s-1} X^{(i)}X^{(i+1)} + Y^{(i)}Y^{(i+1)} + Z^{(i)}Z^{(i+1)}$$





Conclusion



- Block encodings can be easily combined through:
  - Tensor/Kronecker products
  - Linear combinations
- Low-rank tensor decompositions lead to efficient quantum circuits that scale well
- Many problems naturally have (approximate) low-rank tensor structure
  - Localized Hamiltonians
  - Discretized differential operators

Reference: Camps D. and Van Beeumen R., *Approximate quantum circuit synthesis using block encodings*, Phys. Rev. A 102, 052411. DOI:10.1103/PhysRevA.102.052411. arXiv:2007.01417.

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